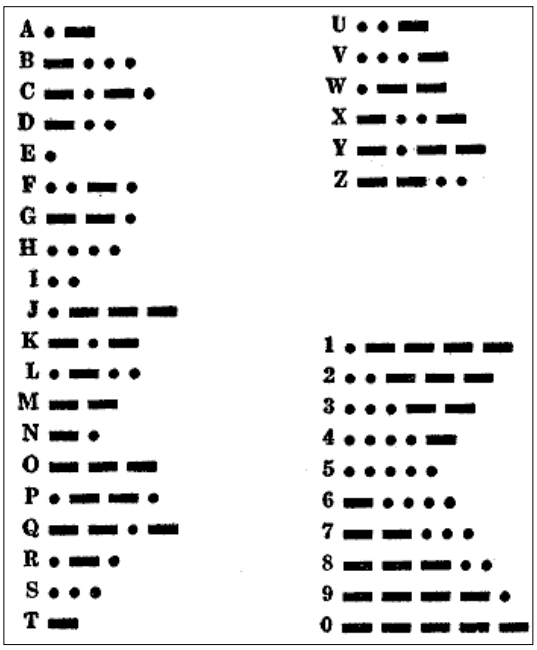
01 Divisibility and Integers

MORSE CODE

This is a chart of the Morse Code letters and numerals.

The rules about its use are the following:

1. A dash is equal to three dots.
2. The space between parts of the same letter is equal to one dot.
3. The space between two letters is equal to three dots.
4. The space between two words is equal to five dots.



Solve the following questions according to the previous rules:

QUESTIONS

Translate into English the following message in Morse code:

Write in Morse code 'You are' using the next plot:

Write in Morse code your name:

Write all the possibilities in order using four signs (dash or dot). For instance,

· · · ·	H						
· · · —	V						
· · — ·	Q						
· — · ·	None						

and so on. You must find sixteen cases.

Locate in the previous chart the related letters to some of the obtained results.

1. DIVISIBILITY

MULTIPLE AND DIVISOR

Multiples of a number are the numbers we get multiplying the number by any natural number. M(2) ={2, 4, 6, 8,… }; M(3) = {3, 6, 9, 12,…}

The multiples of a number are infinite.

A number is divisor of another if the remainder of the division is zero. For example, 5 is a divisor of 15 because 15 = 5 · 3. However, 6 isn't a divisor of 15 because 15 = 6 · 2 + 3. Here the remainder is 3. D(6) = {1, 2, 3, 6}

If a number 'a' is divisor of another 'b' then 'b' is multiple of 'a'. So there is always a relation between divisor and multiple. It is similar to the relation between a father and his son. If ***a*** is father of ***b*** then ***b*** is son of ***a***. For example, 5 is divisor of 15 so 15 is multiple of 5.

DIVISIBILITY RULES

There are some rules about divisibility. We can find out some divisors by these rules. It is a quick way to work out some divisors of a number.

The most common and easy rules are for the numbers: 2, 3 and 5.

Divisible by:	**If:**	**Examples:**
2	The last digit is even (0,2,4,6,8)	12**8** is; 12**9** is not
3	The sum of the digits is divisible by 3	381 (3+8+1=12) **Yes** 217 (2+1+7=10) **No**
4	The last 2 digits are divisible by 4	13**12** is (12÷4=3); 70**19** is not
5	The last digit is 0 or 5	17**5** is; 80**9** is not
6	The number is divisible by both 2 *and* 3	114 **Yes;** 308 **No**
9	The sum of the digits is divisible by 9	1629 **Yes;** 2013 **No**
10	The number ends in 0	22**0** is; 22**1** is not
11	The sum of the even digits minus the sum of the odd digits is either 0 or divisible by 11	1**3**6**4** ((3+4) – (1+6) = **0**) **Yes** 3**7**2**9** ((7+9) – (3+2) = **11**) **Yes** 2**5**1**7**6 ((5+7) – (2+1+6) = **3**) **No**

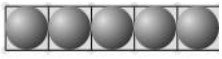
PRIME NUMBERS AND COMPOSITE NUMBERS

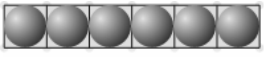
A prime number is the number that has only two divisors: 1 and itself. We can't get the number from a product of smaller numbers. For example 2, 3, 5 are prime numbers.

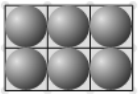
A composite number is the number that has some different divisor than 1 or itself. We can get the number from a product of smaller numbers. For example, 18 is a composite number because it is divisible by 2. That is, 18 = 2 · 9

The prime numbers are called linear numbers because you can made only segment with them. The composite numbers are called rectangular numbers because you can made rectangles with them.

5 is prime number so it is linear number 6 is composite number so it is rectangular number



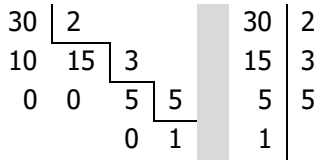




PRIME FACTORIZATION

The prime factorization of a number is the result to put the number as the product of the entire prime factors for the number.

For example, $30 = 2 \cdot 15 = 2 \cdot 3 \cdot 5$



NUMBER OF DIVISORS OF A NUMBER

It is obtained by adding the unit to the exponents and multiplying the obtained results.

How many divisors does the number 30 have? Write all of them in a set.

2. HIGHEST COMMON FACTOR AND LEAST COMMON MULTIPLE

HCF AND LCM

Highest common factor of several numbers is the highest common divisor of them. We represent it by HCF.

Least common multiple of several numbers is the lowest common multiple of them. We represent it by lcm.

CALCULATION OF HCF AND LCM

The HCF is the result of multiplying the prime common factors with the smallest exponent.

HCF (12; 8; 30) =

The lcm is the result of multiplying the entire prime factors with the highest exponent.

lcm (2; 4; 10)=

EUCLIDEAN ALGORITHM

Relationship between the HCF and LCM of *a* and *b*: $\boxed{\text{HCF} \cdot \text{LCM} = a \cdot b}$

Euclid discovered a method useful to calculate the HCF and the lcm of big numbers.

We divide the largest number into the smallest. If the remainder is different to zero we divide the divisor into it and so on. The HCF of two numbers is the last remainder different of zero.

According to these instructions try to calculate the HCF and the lcm for the number 90 and 72.

Calculate by the Euclidean algorithm the HCF and lcm of 630 and 528.

3. INTEGERS

Integers are the natural numbers and their opposite numbers.

This numerical set of numbers is represented by the capital letter Z.

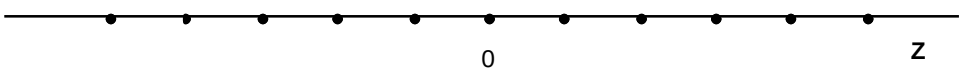
These numbers are infinite in two directions. The set has not an origin and end neither.

Z = {…, –4, –3, –2, –1, 0, 1, 2, 3, 4, …}

REPRESENTATION ON THE NUMERIC LINE

We put the negative numbers from zero to the left in the numerical line, the positive numbers from zero to the right.

Complete the next number line writing the numbers below the point.



ORDER OF INTEGERS

We use the following symbols to represent order relations.

Symbol		Meaning	Example
=	Equality	'Equal to' or equals	8/2=4
≠	Inequality	Different tp	3≠4
<	Strict inequality	Less than	–2<3
>		Greater than	
≤	Inequality	Less than or equal to	
≥		Greater than or equal to	

ABSOLUTE VALUE

Absolute value of a number is the number without the sign. It is the distance between the number and the zero too. E.g.; |–7| = 7; |0| = 0

Two opposite numbers have the same absolute value. |+6| = 6; |–6| = 6

4. INTEGER OPERATIONS

Operations

The rules to operate now are the followings:

To add a negative number is like to add the number. 5+(–3)=5–3

To subtract a negative number is like to add. 5–(–3)=5+3

The product or division of two negative or positive numbers is positive.

The product or division of a negative number and a positive number is negative.

Complete the next chart according to the previous rules:

+ · + = +	+5 · (+ 7) =	+ · – = –	+5 · (– 6) =
– · + = –	–9 · (+ 7) =	– · – = +	–5 · (– 7) =

Order of operations

To do combine operations we have to follow the next order:

First. We do the brackets.

After. Multiplication and division

After. Addition and subtraction.

Operations with same hierarchy we do from [ ] to right.

Example

(3 – 4) · 5 – 2 · (2 – 4 · 3) + 5 =

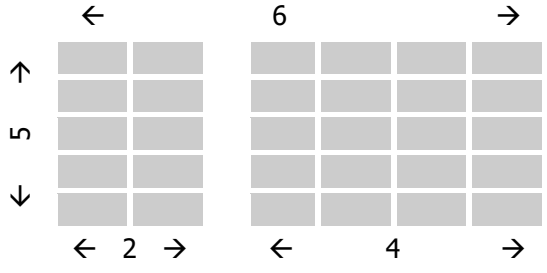
Distributive

The meaning of this property is the following: the result of multiplying an addition or subtraction by a quantity is equal to multiply each term by this quantity.

$$a \cdot (b + c + d) = a \cdot b + a \cdot c + a \cdot d$$

For example, $5 \cdot (2 + 4) = 5 \cdot 2 + 5 \cdot 4$. That is, 5 · 6 = 5·2 + 5·4 = 10 + 20

You have two ways to calculate the number of pupils in a class: multiplying the number of rows by the total of columns or multiplying each part of the class separated by the aisle.



Example:

There is a grant of 3€ per pupil at school. How much does the school need to pay if there are 20 pupils in 2ºA; 23 in 2ºB and 18 in 2ºC. Do the calculation in two different ways.

EXERCISES AND PROBLEMS

1. Divisibility

1. Are the following numbers divisible by 2, 3, 4, 5, 6, 7, 9, 10 and 11? Answer without doing the division.

	2	3	4	5	7	6	9	10	11
924									
1287									
5550									
8725									

2. Change each letter by a digit to get a number divisible by the indicated number. Write all the possibilities:

45a8	**3**
99b	2
21c40	6
4d40	9
ff2	6
1235g	11

3. Calculate all divisors of 120.

4. Calculate all the divisors of the following numbers:
a. 105
b. 220

5. *Write the highest prime number that has two digits and explain why.

6. *Is 391 a prime number? Why?

2. HCF and lcm

7. Calculate the HCF and the lcm of the following numbers by the factorization: 90; 168 and 30.

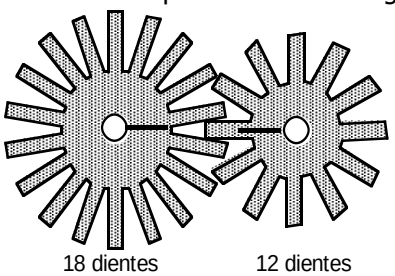
8. Calculate the HCF and the lcm of the following numbers by the factorization: 120; 980 and 110.

9. Calculate the HCF and lcm of the following numbers:
$a = 2^3 \cdot 3^2 \cdot 7^2 \cdot 11;\ b = 2^2 \cdot 3^4 \cdot 5^2 \cdot 7;\ c = 2^3 \cdot 3^3 \cdot 5^2 \cdot 13$

10. Work out the greatest common factor and the least common multiple of the following numbers:
a) 72 y 16
b) 656 y 848
c) 1278 y 842

11. Calculate the greatest common factor and the least common multiple of the following numbers:
a) 72, 108 y 60
b) 1048, 786 y 3930

12. *A gear has 18 teeth and other 12 teeth. How many times does each wheel turn to get the starting position?



13. *Work out the HCF of 258 and 72 by the Euclidean algorithm and the l.c.m. too.

14. *Work out the HCF of 656 and 848 by the Euclidean algorithm and the l.c.m. too.

15. *Calculate the least common multiple and the greatest common factor of 350 and 2695.

16. *Calculate the least common multiple and the greatest common factor of 1617 and 525.

17. *Calculate the least common multiple and the greatest common factor of 1782 and 60984.

3. Integers

18. Calculate:
a) $4 \cdot (10 - 2) - 2 \cdot (-3 + 15) + 9$
b) $8 - [7 - (-2 + 5) - 1] + 4$
c) $5 \cdot (8 - 2 + 3) - (-4) \cdot [6 - (2 + 7)]$

19. Do the following operations:
a) $-4\ (6 - 5) + 6 \cdot (-8) : 4 =$ b) $24 : (5 - 11) - 3\ (25 - 30) =$

20. Do the following operations:
a) $-3\ (2 - 7) + 5 \cdot (-6) : 3 =$ b) $16 : (4 - 12) - 3\ (25 - 30) =$

21. Calculate:
$(7 - 2 + 4) - (2 - 5) =$
$1 - (5 - 3 + 2) - [5 - (6 - 3 + 1) - 2] =$
$-12 \cdot 3 + 18 : (-12 : 6 + 8) =$

22. Complete with a number:
$a) -5 + \quad = -12 \quad b) -6 + \quad = 15 \quad c) \quad + (-7) = -5 \quad d) \quad + (-4) = 9$
$e) 9 - \quad = -9 \quad f) -7 - \quad = -7 \quad g) \quad - (-8) = -5 \quad h) 3 - \quad = 9$

23. Calculate:
$(3 - 8) + [5 - (-2)] =$
$5 - [6 - 2 - (1 - 8) - 3 + 6] + 5 =$
$9 : [6 : (-2)] =$
$[(-2)^5 - (-3)^3]^2 =$

24. Calculate step by step: $(5 + 3 \cdot 2 : 6 - 4) \cdot (4 : 2 - 3 + 6) : (7 - 8 : 2 - 2)^2 =$

25. Calculate step by step: $[(17 - 15)^3 + (7 - 12)^2] : [(6 - 7) \cdot (12 - 23)] =$

26. Find the missing number in each calculation:

a	$2 \times -3 = \square$	**b**	$-2 \times \square = -8$	**c**	$3 \times \square = -9$
d	$\square \div -5 = -15$	**e**	$-4 \times -6 = \square$	**f**	$-3 \times \square = -24$
g	$-64 \div \square = 32$	**h**	$\square \times 6 = 36$	**i**	$-2 \times 3 = \square$
j	$\square \times -6 = -48$	**k**	$-2 \times \square \times 3 = 12$	**l**	$\square \div -4 = 2$
m	$5 \times 4 \div \square = -10$	**n**	$-5 \times \square \div -2 = -10$	**o**	$\square \times -4 \div -2 = 14$

27. Find and draw all the integers that work: $-4 \leq x < 5$

28. Find and draw the integer numbers that work: $|x| < 4$

29. Find and draw all the integer numbers that fit the following property: $-7 < x \leq -1$

30. Find and draw the integer numbers that work $|x| \leq 3$

31. The temperature in a room is 25ºC in winter. We stop the heating and thus it goes down 5ºC per hour. What's the temperature 6 hours later? Do the calculation in a mathematical way.

32. A Roman emperor was born in 63 B.C. and died in 14 A.D. How many years did he live? Do the calculation in a mathematical way.

33. A bomb displaces the oil in a well from 975 m deep and raises it to a tank places 48 m above ground level. How far is the oil raised? Do the calculation in a mathematical way.

34. *If 'b' is a negative number, what sign have the following mathematical expressions?:

a) $-2 \cdot b$

b) $-2 \,.\, (-3) \,.\, (-b)$

c) $-2 \cdot (1 - b)$

d) $-3 \cdot (b - 1)$